Electrostatics

* Forces & static interactions of electrically charged particles *

Begin by jumping into some examples, break it down throughout.

Coulomb's Law

Describes force acting on charge from other charges in the system/nearby

* Like charges repel, opposites attract * ← important

$$F = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

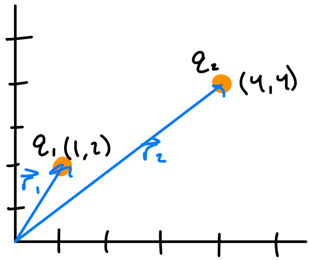
q_1 - charge of particle 1
q_2 - charge of particle 2
k - constant
r_{12} - distance between charges
$\hat{r}_{12}$ - unit vector from point 1 to point 2

double check this

Formula describes force exerted on charge 2 by charge 1

Note on Notation: $F_{12} \Rightarrow$ Force exerted on charge 2 by charge 1

* unit vector does not indicate direction of force on it's own
↳ need to consider sign of each charge
↳ * just line along which force is exerted



Location/position of each particle can be denoted w/ vector

$$\vec{r}_1 = \langle 1, 2 \rangle = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{r}_2 = \langle 4, 4 \rangle = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

Q: Official notation for position of particle?
↳ specific variable I should use?
*I think r.

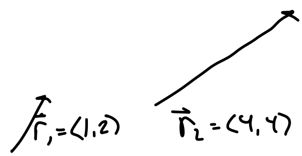
Vector pointing from q_1 to q_2 is $\vec{r}_{12}$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$= \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

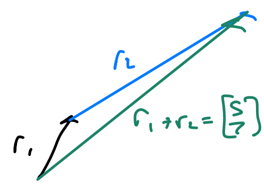
Arithmetic is important...

Quick note on vector arithmetic.



add 2 vectors → tip to tail

$$\vec{r}_1 + \vec{r}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$



tip of r_2 to tail of r_1

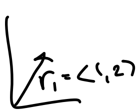
* same if you do $r_2 + r_1$ → tip of r_1 to tail of r_2

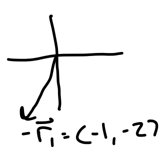
Subtract 2 vectors → add the negative

$$\vec{r}_2 - \vec{r}_1 = \vec{r}_2 + (-\vec{r}_1)$$ - equivalent expressions

This is the way I like to do vector subtraction

negative of a vector just flips it

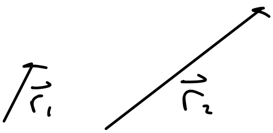




so subtracting is adding the flipped vector

$$\vec{r}_2 - \vec{r}_1 = \langle 4,4 \rangle - \langle 1,2 \rangle = \langle 3,2 \rangle$$

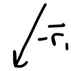
Adding the negative is more for visualizing vector subtraction



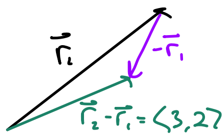
$\vec r_2 - \vec r_1 = \vec r_2 + (-\vec r_1)$

First we find the negative $\vec r_1$

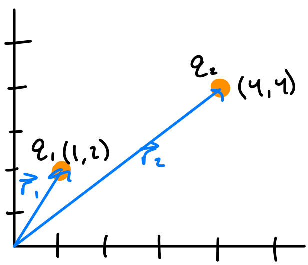
$-\vec r_1 = \langle -1, -2 \rangle$



Then add to $\vec r_2$, tip to tail



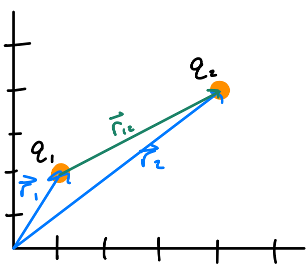
Returning to electrostatics problem...



$*\ \vec r_2 - \vec r_1 = \vec r_{12}$

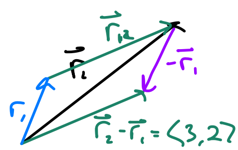
$\vec r_{12}$ points from $\vec r_1$ to $\vec r_2$

↳ force from q_1 acting on q_2



Take result of vector subtraction & position it accordingly

↳ * i.e. it does NOT start at origin



← $\vec r_{12}$ begins at $\vec r_1$ (tail) & ends at tail of $\vec r_2$

Length & Unit Vector

$|\vec r_{12}|$ - magnitude/length of vector $\vec r_{12}$

$\hat r_{12}$ - unit vector pointing in direction of $\vec r_{12}$ (from $\vec r_1$ (q_1) to $\vec r_2$ (q_2))

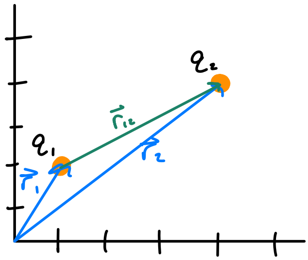
Magnitude: $|\vec{V}| = \sqrt{V_1^2 + V_2^2}$

Unit Vector: $\hat{V} = \frac{\vec{V}}{|\vec{V}|}$

Since we are working in 2-d space

Generally, if $\vec{V} \in \mathbb{R}^n$ then

$|\vec{V}| = \sqrt{V_1^2 + V_2^2 + \cdots + V_n^2}$



Consider...

$q_1 = +1\mu C$

$q_2 = +2\mu C$

$\vec{r}_{12} = \langle 3, 2 \rangle$

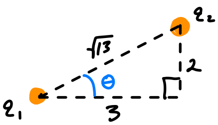
$|\vec{r}_{12}| = r_{12} = \sqrt{(3)^2 + (2)^2} = \sqrt{13}$

Could've picked better example

Coulomb Constant

$k = 8.99 \times 10^9 \ \frac{N \cdot m^2}{C^2}$

* Force acting in 2 directions so need to find x & y components



$F_{12_x} \rightarrow$ $F_x = F_{12} \cos\theta$

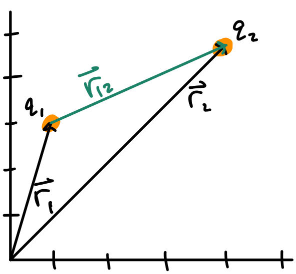
$F_y = F_{12} \sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Good thing I'm starting from section 1. I don't have the fundaments down as well as I thought.

→ Going to walk through what I believe correct process is before checking notes. Going to do more formalized problem.

System consists of 2 charges. $q_1 = +2\mu C$ $q_2 = +3\mu C$.
Position of each particle is: $\vec{r}_1 = \langle 1, 3 \rangle$ $\vec{r}_2 = \langle 4, 5 \rangle$.
Find magnitude of force between the two charges & direction of force each exerts on the other.



Magnitude of force only requires distance between particles & is also necessary (I believe) for calculating F_x & F_y

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = \langle 4, 5 \rangle - \langle 1, 3 \rangle = \langle 3, 2 \rangle$

$|\vec{r}_{12}| = r_{12} = \sqrt{(3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$

$$|\vec{F}_{12}| = F_{12} = \frac{k|q_1 q_2|}{r_{12}^2}$$

↙ notice MAGNITUDE of force doesn't care about sign of charges or unit vector.

* Magnitude of electric static force is proportional to magnitude of product of 2 charges (larger magnitude charge → larger magnitude force)

* $|\vec{F}_E|$ inversely proportional to distance between 2 charges

related?

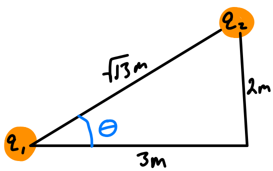
$$F_{12} = \frac{k|q_1 q_2|}{r_{12}^2} = \frac{\left(8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}\right)\left(2.0\times10^{-6}\,C\right)\left(3.0\times10^{-6}\,C\right)}{\left(\sqrt{13}\,m\right)^2}$$

$$= 0.0041\ N$$

$$F_{12} = 4.1\times10^{-3}\ N$$

Magnitude of force should always be positive.

Now break down component-wise



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

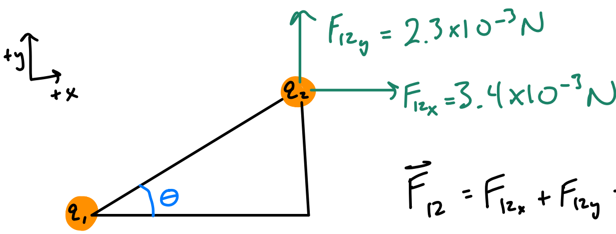
$$= \tan^{-1}\left(\frac{2}{3}\right)$$

$$\theta \approx 33.7^\circ$$

*make sure calculator is in degrees

$$F_{12x} = F_{12}\cos\theta = \left(4.1\times10^{-3}\,N\right)\cos(33.7^\circ) = 0.0034 = 3.4\times10^{-3}\,N$$

$$F_{12y} = F_{12}\sin\theta = \left(4.1\times10^{-3}\,N\right)\sin(33.7^\circ) = 0.00227 = 2.3\times10^{-3}\,N$$



← both x & y force in positive direction

$$\vec{F}_{12} = F_{12x} + F_{12y} = \left(2.3\times10^{-3}\right)\hat{\imath} + \left(3.4\times10^{-3}\right)\hat{\jmath}$$

*Note - if q_1 & q_2 had opposite signs they would be attracted to one another.

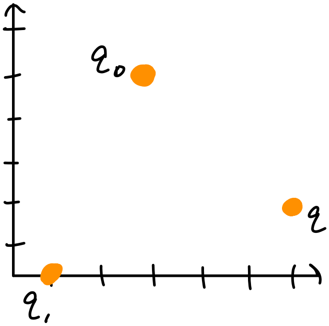
Magnitude of force would be same but direction would be different

Could calculate direction of force on q_1 but going to move on instead

Now consider a system of point charges. 3 or more

***Superposition Principle** – (in context of Coulomb's Law)

↳ Total force $\vec{F}_{net}$ acting on a particle is the sum of individual forces from other charges in system



Consider system of point charges.

$q_0 = +20nC \quad q_1 = -30nC \quad q_2 = +25nC$

$\vec{r}_0 = \langle 3, 5\rangle \quad \vec{r}_1 = \langle 1, 0\rangle \quad \vec{r}_2 = \langle 6, 2\rangle$

Find total force, $\vec{F}_{net}$, exerted on q_0

$$\vec{F}_{net} = \vec{F}_{10} + \vec{F}_{20}$$

← superposition principle

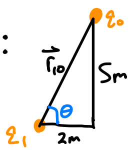
For charge q_0 in system w/ n other charges

Total force on q_0 is sum of individual forces

$$\vec{F}_{net} = \sum_{i=1}^{n} \vec{F}_{i0}$$

Calculate each force individually

$\vec{F}_{10}$:



$\vec{r}_{10} = \vec{r}_0 - \vec{r}_1 = \langle 3, 5\rangle - \langle 1, 0\rangle = \langle 2, 5\rangle$

$|\vec{r}_{10}| = r_{10} = \sqrt{2^2 + 5^2} = \sqrt{29}$

$\theta = \tan^{-1}\left(\frac{5}{2}\right) \simeq 68.2°$

Don't need absolute value

* Shouldn't have it

$$|\vec{F}_{10}| = F_{10} = \frac{k|q_0 q_1|}{r_{10}^2} = \frac{(8.99\times10^9)|(20\times10^{-9})(-30\times10^{-9})|}{(\sqrt{29})^2} = 1.86\times10^{-7}N$$

$$F_{10x} = F_{10}\cos\theta = (1.86\times10^{-7}N)\cos(68.2°) = 6.91\times10^{-8}$$

$$F_{10y} = F_{10}\sin\theta = (1.86\times10^{-7}N)\sin(68.2°) = 1.73\times10^{-7}$$

This gives magnitude but we can adjust to get direction as well

Unit vector gives direction

$$\vec{F}_{10} = \frac{k q_0 q_1}{r_{10}^2}\hat{r}_{10} \qquad \hat{r}_{10} = \frac{\vec{r}_{10}}{|\vec{r}_{10}|} = \cancel{\left\langle \frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\right\rangle = \left(\frac{2}{\sqrt{29}}\right)\hat{\imath} +}$$

* Need to incorporate direction right away. I know q_0 is (+) & q_1 is (-) so force on q_0 will be TOWARDS q_1. So both x & y components of force should be negative.

$$\hat{r}_{10} = \left(-\frac{2}{\sqrt{29}}\right)\hat{\imath} + \left(-\frac{5}{\sqrt{29}}\right)\hat{\jmath} = \left\langle -\frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}}\right\rangle$$

$$\vec{F}_{10} = \left[\frac{k q_0 q_1}{r_{10}^2}\right]\left[\left(-\frac{2}{\sqrt{29}}\right)\hat{\imath} + \left(-\frac{5}{\sqrt{29}}\right)\hat{\jmath}\right]$$

*Could have left $\hat{r}_{10}$ (+) & had force negative?

$$= (1.86 \times 10^{-7}\,N)\left[\left(-\frac{2}{\sqrt{29}}\right)\hat{\imath} + \left(-\frac{5}{\sqrt{29}}\right)\hat{\jmath}\right]$$

$$\vec{F}_{10} = (-6.91 \times 10^{-8})\hat{\imath} + (-1.73 \times 10^{-7})\hat{\jmath}$$

same magnitude as before only we now included correct direction

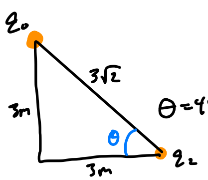
q_1 exerts $6.91 \times 10^{-8}\,N$ of force on q_0 in negative x-direction
$1.73 \times 10^{-7}\,N$ of force on q_0 in negative y-direction

Confirm total magnitude checks out

$$F_{10} = \sqrt{F_{10_x}^2 + F_{10_y}^2} = \sqrt{(6.91 \times 10^{-8})^2 + (1.73 \times 10^{-7})^2} = 1.86 \times 10^{-7}\,N \checkmark$$

*KEY TAKEAWAY → MAKE SURE YOU PAY ATTENTION TO SIGNS

$\vec{F}_{20}$:



$$\vec{r}_{20} = \vec{r}_0 - \vec{r}_2 = \langle 3,5\rangle - \langle 6,2\rangle = \langle -3, 3\rangle$$

$$\hat{r}_{20} = \frac{\vec{r}_{20}}{|\vec{r}_{20}|} = \frac{\langle -3, -3\rangle}{3\sqrt{2}} = \left(-\frac{1}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{1}{\sqrt{2}}\right)\hat{\jmath}$$

I think doing $F = \frac{k q_0 q_1}{r_{10}^2}\hat{r}_{10}$ w/ no abs values & original unit vector would make it easier

$$\vec{F}_{20} = \frac{k q_0 q_2}{r_{20}^2}\hat{r}_{20}$$

$$= \left[\frac{(8.99 \times 10^9)(20 \times 10^{-9})(25 \times 10^{-9})}{(3\sqrt{2})^2}\right]\left[\left(-\frac{1}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{1}{\sqrt{2}}\right)\hat{\jmath}\right]$$

$$= (2.497 \times 10^{-7})\left[\left(-\frac{1}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{1}{\sqrt{2}}\right)\hat{\jmath}\right]$$

$$\vec{F}_{20} = (-1.77 \times 10^{-7})\hat{\imath} + (1.77 \times 10^{-7})\hat{\jmath}$$

Can confirm by looking at diagram. Both q_0 & q_2 are positive so force on q_0 will be away from q_2 so in negative x & positive y directions

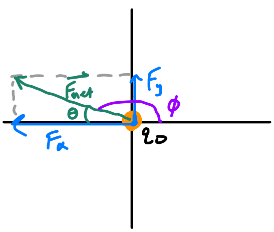
$\cancel{\vec{F}_{net} = F_x + F_y}$

$\cancel{\vec{F}_x = \vec{F}_{10x} + \vec{F}_{20x}} = \left[(-6.91\times10^{-8})\hat{i} + (-1.73\times10^{-7})\hat{j}\right] + \left[(-1.77\times10^{-7})\hat{i} + (1.77\times10^{-7})\hat{j}\right]$

$\vec{F}_{net} = \vec{F}_{10} + \vec{F}_{20}$

$$\boxed{\vec{F}_{net} = (-2.46\times10^{-7})\hat{i} + (4.0\times10^{-9})\hat{j}}$$

Can find magnitude & direction (angle)



$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-2.46\times10^{-7})^2 + (4.0\times10^{-9})^2}$$
$$= 2.46\times10^{-7}$$

$$\theta = \tan^{-1}\left(\frac{4.0\times10^{-9}}{-2.46\times10^{-7}}\right) \simeq -0.93°$$

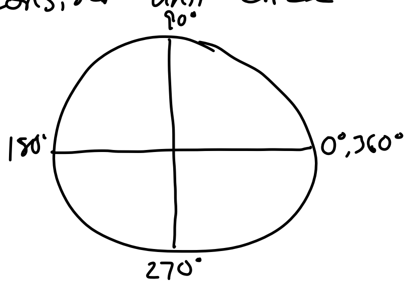
This is part I feel I make mistakes, but it does checkout
Consider angle ϕ measuring direction of force from +x direction

$\phi = 180 - \theta = 180.93$

Okay nevermind. My trig is a bit rusty but I'm going to break this down

I guess θ is being measured in opposite direction

Consider unit circle



Degrees increase as you go counterclockwise
So considering −x axis as 0 & measuring θ clockwise should result in (−) angle

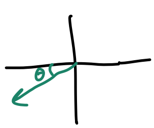
I guess my question is just how I should present my answer

& I just need to be aware the signs of θ can be weird.

> *Important thing is I am explicit & stay consistent w/ which directions I indicate as positive

* Should have been $\phi = 180 + \theta$ not $-\theta$

If force vector looked like



$\tan^{-1}\left(\frac{-y}{-x}\right) = +\theta$

result would have been greater than 180°

Can say 0.93° above −x axis
or 179.07° from +x axis
(standard unit circle convention)