Electric Fields

Where does force between charges come from?

Electric field analogous w/ gravitational fields
Only for charges instead of matter

* Any charged object produces an electric field that propagates through space.
  - This field is what exerts a force on other charged objects nearby

Coulomb's Law (Electrostatic force):

$$\vec{F}_E = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Electric field at any point in space is the electrostatic force experienced by a charge at that point.

* We can calculate field at point w/ no real charge by considering the force a positive test charge, q_0, would experience at that point

$$\vec{E} = \frac{\vec{F}}{q_0}$$

* SI units – N/C *

Force / Charge

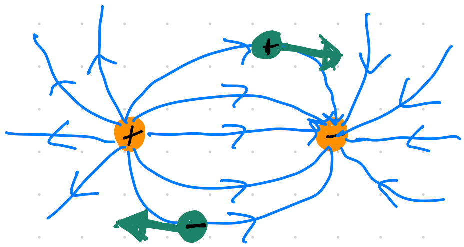
Particle has charge $-3.35nC$

(a) Magnitude & direction of electric field point 0.20m above it?

(b) Find distance from particle where magnitude of field is $11.2\ N/C$

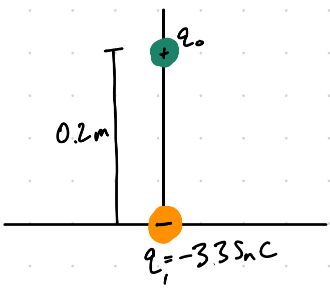
Positive/Negative Charges

Electric fields "travel" in certain direction
From (+) charge to (-) charge



Place a (+) & (-) test charge in field
- (+) travels with field
- (-) travels against field

Consider problem above



Unless told otherwise, measure $\vec{E}$ using + test charge q_0
where q_0 is infinitely small
↳ no $\vec{E}$ from q_0 to effect system

$$\vec{E} = \frac{\vec{F}_{10}}{q_0}$$

$\vec{F}_{10}$ is Coulomb's Law.
Force exerted on q_0 by q_1

$$\vec{F}_{10} = \frac{k q_1 q_0}{r_{10}^2} \hat{r}_{10}$$

Plug $\vec{F}_{10}$ into eq.

$$\vec{E} = \frac{\frac{k q_1 q_0}{r_{10}^2} \hat{r}_{10}}{q_0} = \frac{k q_1}{r_{10}^2} \hat{r}_{10}$$

* Field at point P is dependent on magnitude of charge & distance from charge *

$\vec{r}_{10} = \vec{r}_0 - \vec{r}_1 = \langle 0, 0.2\rangle - \langle 0, 0\rangle$

$\vec{r}_{10} = \langle 0, 0.2\rangle$

$|\vec{r}_{10}| = 0.2\,m$ we knew distance already

$\hat{r}_{10} = \frac{\vec{r}_{10}}{|\vec{r}_{10}|} = \frac{\langle 0, 0.2\rangle}{0.2}$

$= 0\hat{\imath} + 1\hat{\jmath}$

$= +\hat{\jmath}$ Force experienced by test charge is purely in y-direction

$$\vec{E} = \frac{kq_1}{r_{10}^2}\hat{r}_{10} = \left[\frac{(8.99\times10^9)(-3.35\times10^{-9})}{(0.2)^2}\right]\hat{\jmath}$$

$= -752.9\,\hat{\jmath}$

$= (-7.5\times10^2\,N/C)\,\hat{\jmath}$

Positive charge 0.2 m above q_1 experience 7.5×10^2 N of force straight down per Coulomb of charge.

Use this to find force experienced by different charges at this point.

New charge $q_2 = +2.5\,nC$ at same point. What is force?

$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = \vec{E}q$

$\vec{F}_{12} = (-7.5\times10^2\,N/C)(2.5\times10^{-9}\,C)$

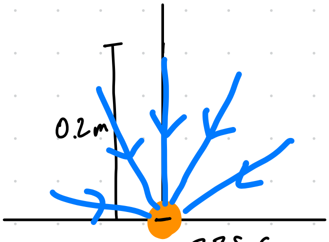
$= -1.86\times10^{-6}\,N$ Force on positive charge down

$q_3 = -2.5\,nC$

$\vec{F}_{13} = (-7.5\times10^2)(-2.5\times10^{-9})$

$= 1.86\times10^{-6}\,N$ Force on negative charge is up

Looking at system again



* Field produced by (-) charge points TOWARDS charge

(+) charge travels w/ field lines
(-) charge travels opposite field lines

Demonstrated this above

(b) At what distance is magnitude of $\vec{E}$ 11.2 N/C?

Can ignore $\hat{r}_{10}$ since focused purely on magnitude

$$\vec{E} = \frac{kq_1}{r_{10}^2}$$

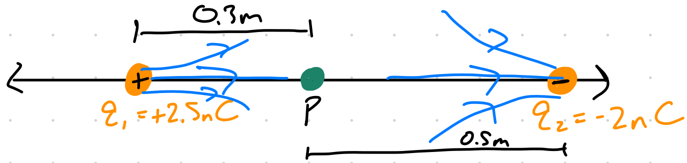
$$|\vec{E}| = \frac{k|q_1|}{r_{10}^2} \Rightarrow r_{10}^2 = \frac{k|q_1|}{E} \Rightarrow r_{10} = \sqrt{\frac{k|q_1|}{E}}$$

$$r_{10} = \sqrt{\frac{(8.99\times10^9)|(-3.35\times10^{-9})|}{11.2}}$$

$$\approx 1.64\,m$$

* Superposition principle applies to electric fields

if there are multiple charges producing fields in a system, Electric field at point P is sum of individual fields



We can see direction of $\vec{E}$ immediately

$$\vec{E}_p = \vec{E}_{1p} + \vec{E}_{2p}$$ ← Total field at P is sum of fields from charge 1 & 2 at point P

$$\vec{E}_{1p} = \frac{kq_1}{r_{10}^2}\hat{r}_{10} = \left[\frac{(8.99\times10^9)(2.5\times10^{-9})}{0.3^2}\right]\hat{\imath}$$

$$\hat{r}_{10} = \frac{\langle .3, 0\rangle}{.3} = +\hat{\imath} + 0\hat{\jmath}$$

$$= (250\ N/C)\hat{\imath}$$

$$\vec{E}_{2p} = \frac{kq_2}{r_{20}^2}\hat{r}_{20} = \left[\frac{(8.99\times10^9)(-2.0\times10^{-9})}{(0.5)^2}\right](-\hat{\imath})$$

$$\hat{r}_{2p} = \frac{\langle -0.5, 0\rangle}{0.5} = -\hat{\imath}$$

negatives cancel

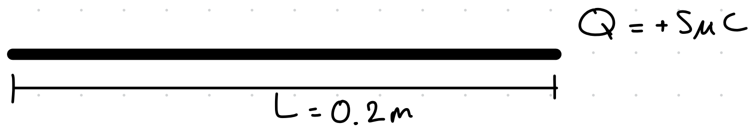
$$= (71.9\ N/C)\hat{\imath}$$

$$\vec{E}_{net} = (250)\hat{\imath} + (71.9)\hat{\imath} = (321.9\ N/C)\hat{\imath}$$

if q_2 was positive, the force on positive charge at P would be to left. Indicated by unit vector $-\hat{\imath}$

What about charged objects other than point particles?

Consider a metal rod w/ a charge distributed uniformly across it. (Positive charge)



When calculating field at any point P, we need to factor in field produced by entire rod, which will have different distances & directions to P.

* Superposition principle *

↳ Sum up field at P produced by every (infinitesimally small) segment of the rod.

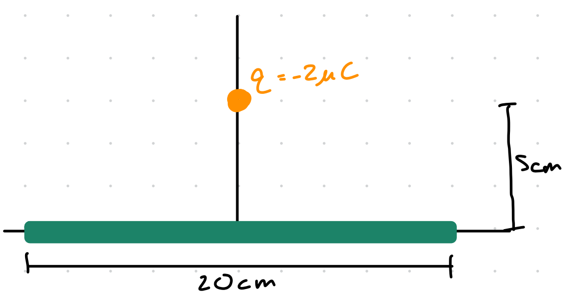
Charge of each segment is charge per unit length

$\lambda = \frac{Q}{L}$

Uniform line charge length 20.0cm on x-axis w/ midpoint @ $x=0$ & $\lambda = +4.80\,nC/m$

Small sphere $q = -2\mu C$ placed at $x=0$ $y=5cm$.

What $\vec{E}$ does sphere exert on line of charge?



$\lambda = 4.80\,nC/m$

Segment of line, dL

Find field from sphere at that segment $d\vec{E}$

$\vec{E}$ is sum of $d\vec{E}$ for each segment dL

$$\boxed{\vec{E} = \int_{\text{line}} d\vec{E}}$$

$$\lambda = \frac{Q}{L} \Rightarrow Q = \lambda L$$

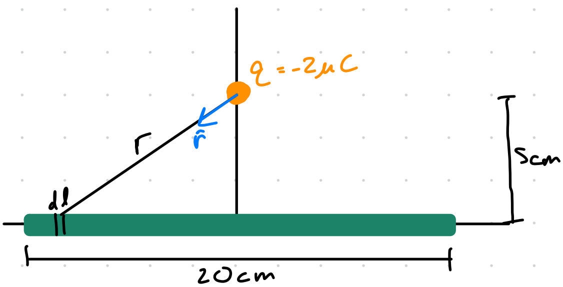
charge in segment dl of line is dQ

$$dQ = \lambda \, dl$$

Same field formula as before, only for $d\vec{E}$

$$d\vec{E} = k \frac{dQ}{r^2} \hat{r}$$

dQ is charge of segment dl
r distance from dl to P
$\hat{r}$ direction
$d\vec{E}$ field of P from segment dl



* $\hat{r}$ points from SOURCE of field to where it is being measured

This one's different than I've done in past. Usually finding field FROM line charge. Should be same principle
Going to play around w/ it a bit

$$\vec{E} = \frac{\left[\frac{k q_1 q_p}{r^2} \hat{r}\right]}{q_p}$$

in this example, P is every segment of line

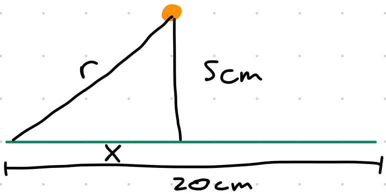
Field from sphere on each line segment

$$\frac{\frac{k q_1 \, dQ}{r^2} \hat{r}}{dQ}$$

I think λ is irrelevant in this problem

Intuitively I have an idea for answer
$\vec{E}_x$ from left & right of origin will cancel & some field purely in y

$\frac{kq}{r^2}\hat{r}$



$r = \sqrt{x^2 + 5cm^2}$

At any point x along line
$\vec{r} = \langle x, 0\rangle - \langle 0, 5cm\rangle$
$\vec{r} = \langle x, -5cm\rangle$ where x is position in interval (-10, 10)

$$d\vec{E} = \frac{kq}{r^2}\hat{r} = \left[\frac{kq}{x^2+.05^2}\right]\left[(x)\hat{i} + (-0.05)\hat{j}\right]$$

think my $\hat{r}$ is wrong

$$\vec{E} = \int_{-0.1}^{0.1}\left[\frac{kq}{x^2+0.0025}\right]\left[(x)\hat{i} - 0.05\hat{j}\right]dx$$

← evaluate from left endpoint (-10cm) to right (10cm)

Separate E_x & E_y?

$$\vec{E}_x = \int_{-0.1}^{0.1}\frac{kqx}{x^2+0.0025}\,dx\ \hat{i}?\qquad E_y = \int_{-0.1}^{0.1}\frac{-0.05kq}{x^2+0.0025}$$

k & q are constants
Going to ignore the x component for now since I know it will cancel. Would like to be confident in proving that w/ the math though

$$\vec{E} = -0.05kq\int_{-0.1}^{0.1}\frac{1}{x^2+0.0025}\,dx$$

Now I show how badly I need to practice integrals

Might have set it up wrong too

Correct $\hat{r}$

$\vec{r} = \langle x, 0\rangle - \langle 0, 5cm\rangle$
$= \langle x, -5cm\rangle$

$|\vec{r}| = \sqrt{x^2 + 5cm^2}$

$$\hat{r} = \left(\frac{x}{\sqrt{x^2+5^2}}\right)\hat{i} + \left(\frac{-5}{\sqrt{x^2+5^2}}\right)\hat{j}$$

$$d\vec{E} = \frac{kq}{r^2}\hat{r}$$

* Convert from cm to m at end

$$d\vec{E} = \left[\frac{kq}{x^2+25}\right]\left[\left(\frac{x}{\sqrt{x^2+25}}\right)\hat{i} + \left(\frac{-5}{\sqrt{x^2+25}}\right)\hat{j}\right]$$

Distribute & separate integrals

$$\vec{E} = \int d\vec{E} = \int \left[\frac{kq}{x^2+25}\right]\left[\left(\frac{x}{\sqrt{x^2+25}}\right)\hat{\imath} + \left(\frac{-5}{\sqrt{x^2+25}}\right)\hat{\jmath}\right] dx$$

$$= \int \frac{kqx}{(x^2+25)^{3/2}}\,dx\ \hat{\imath} + \int \frac{-5kq}{(x^2+25)^{3/2}}\,dx\ \hat{\jmath}$$

Start w/ $\hat{\imath}$ to show $E_x = 0$, if it does

$$kq\int_{-10}^{10} \frac{x}{(x^2+25)^{3/2}}\,dx \qquad u = x^2+25$$

$$du = 2x\,dx$$

$$\tfrac{1}{2}du = x\,dx \quad \leftarrow \text{factor out } \tfrac{1}{2}$$

$$= \frac{1}{2}kq\int_{-10}^{10}\frac{1}{u^{3/2}}\,du \qquad \int u^{-3/2}\,du = \cancel{-\tfrac{1}{2}u^{-1/2} = -\frac{1}{2u^{1/2}}}$$

$$= \frac{u^{-1/2}}{-\frac{1}{2}} = -2u^{-1/2} = -\frac{2}{u^{1/2}}$$

think correct up to here

$$\cancel{= \frac{1}{2}kq\left[-\frac{x}{2(x^2+25)^{3/2}}\right]_{-10}^{10} = -\frac{10}{2(10^2+25)^{3/2}} - \Big(}$$

$$= \frac{1}{2}kq\left[\frac{-2x}{\sqrt{x^2+25}}\right]_{-10}^{10}$$

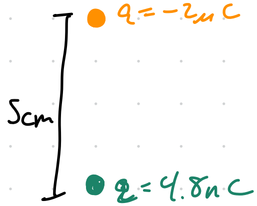
$$= -kq\left[\frac{x}{\sqrt{x^2+25}}\right]_{-10}^{10} \Rightarrow \frac{10}{\sqrt{10^2+25}} - \left[\frac{-10}{\sqrt{10^2+25}}\right.$$

Maybe E_x doesn't cancel to 0. I don't see how that would be. think I'm making a mistake somewhere

Actually I think I may have been doing problem wrong. Or set it up wrong. Asking for FORCE on line from sphere

$$\vec{F} = \vec{E}q$$

Definitely much easier than I'm making it.
X cancels. dl at origin is only one that matters
q of dl is 4.8 nC
Just look at it as point charge



$$F = \frac{k q_1 q_2}{r^2}\hat{r}$$

$$\vec{r} = (0,0) - (0,5)$$
$$= (0,-5)$$
$$\hat{r} = -\hat{j}$$

$$= \frac{(8.99\times10^{9})(-2\times10^{-6})(4.8\times10^{-9})}{0.05^2} - \hat{j}$$

$$= 0.0345 \approx 3.5\times10^{-3}\,N \text{ in positive } y$$

Solution in book was 3.09×10^{-3}

$$q = \lambda L$$
$$= 4.8\,(.02)$$
$$q = 0.096\,nC = 9.6\times10^{-10}$$

Don't think I need to do this answer was even more incorrect

Check what book uses for value of k.

$k = 8.988$

didn't make any difference

Do need to do full integral
Setting it up different this time

$$\vec{F} = \vec{E}q$$
$$d\vec{F} = d\vec{E}\,dq = \frac{k q\, dq}{r^2}\hat{r} = kq\left[\frac{\lambda dx}{r^2}\hat{r}\right] = \lambda k q \frac{dx}{r^2}\hat{r}$$

$$\vec{F} = \lambda k q \int \frac{dx}{r^2}\hat{r}$$

Might call it for today & do this one tomorrow.